Entropy statistics as a methodology to analyse the evolution of complex technological systems. Application to aircraft, helicopters and motorcycles[•]

Prepared for the European Meeting on Applied Evolutionary Economics
Grenoble, France, 7-9 June 1999

Koen Frenken

Department of Science and Technology Dynamics, University of Amsterdam
Nieuwe Achtergracht 166, 1018 WV Amsterdam, The Netherlands
Tel: 00.31.20.525.6558, Fax: 00.31.20.525.6579, email: kfrenken@chem.uva.nl

Abstract: A model of technological evolution is developed on the basis of Kauffman's NK-model. This model takes into account the complex interdependencies between a technology's components. The complexity of a technology implies that several design solutions are more or less equally fit (these correspond to local optima in a fitness landscape). These design solutions are based on different trade-offs between functions of a technology and characterise possible technological trajectories. In which design solution a firm may end up is path-dependent on its search history. After discussing specific features of technological evolution absent in biological evolution (imitation, dynamic efficiencies), it is argued that the NK-model is a candidate for a general model of market differentiation through product innovation. The model is applied to data on technical characteristics of aircraft (1913-1984), helicopters (1940-1983) and motorcycles (1911-1995) using entropy statistics. We test two hypothesis: (i) increasing technological complexity increases the scope for differentiation, and (ii) increasing technological complexity increases the scope for specialisation of firms.

Key-words: NK-model, complexity, technological evolution, local optima, technological trajectory, market differentiation, path-dependence, entropy statistics

[•] The research reported here is partially based on data gathered in a project funded by the ESRC (Saviotti 1996).

The notion of evolution is economic theorising has been persuasive in understanding the competition process between technologies. The static concept of equilibrium in neoclassical theory can be understood in dynamic terms: competing firms tend to converge to the optimal technology through a process of selection. This evolutionary argument in economics follows the "The Fundamental Theorem of Natural Selection" in biology which states that the change in the relative share of a particular genotype within a population is proportional to its fitness relative to the fitness of other genotypes present in the population (Fisher, 1930). The economic analogue of natural selection holds that the rate of expansion of a technology is proportional to its relative efficiency, as firms adopting a more efficient technology are expanding their output at a higher rate than firms adopting a less efficient technology (Alchian, 1950).

Models of technological competition usually deal with selection dynamics of competing technologies, which can take different shapes according to the structure of returns and market shares. In these models, technology is still treated as a "black box" solely characterised by its efficiency. Consequently, these models are limited to the analysis of cost competition either through increasing returns (Arthur 1989) or process innovation (Nelson and Winter 1982). Relatively few models in economics deal with the qualitative evolution of technology and the emergence of new market niches through product differentiation. Here, a model is proposed that is based on a description of technologies in terms of a set of components and their trade-offs. Each technological design then, can be represented as a combination of components. This combinatorial, evolutionary model includes the possibility of new variety to be created by combining existing components and developing of new components.

Following the NK-model (Kauffman, 1993), we represent technologies as *complex* systems that contain a set of components that function interdependently. By combining different components, firms search the space of possible designs. Complex technologies are characterised by local optima, which are combinations between components that are superior to combinations that differ in one component. These optima reflect different compromises between conflicting design constraints, thus embodying different trade-offs between its functions. As a firm myopically searches the combinatorial space, it is expected to lock into a local optimum. As different optima embody different trade-offs, a firm can be expected to specialise in the market segment to which these trade-offs correspond. However, the *industry* need not to lock into one local optimum, since different firms may end up in different local optima. We then can start to analyse to what extent firms specialise in product development of one specific design, and which global pattern emerges at the industry level.

The paper is organised as follows. First, the model of technological evolution is developed using NK-model. Then, an empirical methodology based on entropy statistics is developed which is applied to time-series on product characteristics of aircraft, helicopters and motorcycles. The results will be related to the NK-model and compared with patterns in industrial evolution in terms of the number of firms. Concluding remarks are listed in the final section.

1. Complex technological systems

The NK-model simulates the evolution of complex systems that are represented by a string of elements. The NK-model has been primarily developed to analyse populations of organisms that are described by a string of genes, but its formal structure allows for applications in other domains (Kauffman, Macready, 1995; Levinthal, 1997; Frenken, 1998; Marengo, 1998). In the context of the evolution of technological artefacts, the elements of a string can be taken to refer to the components incorporated in the technology. The variable N then refers to the number of components that describe a product ($z=1,\ldots,N$). For example, an aircraft can be described by its engine, wing, material, cooling device, etc. To design an artefact, firms choose for each component among a number of variants of this component (alleles). Following the aircraft example, components variants are a propeller engine or a jet engine, a swept wing or delta wing, metal or wood material, air- or water-cooling, etc. The number of all possible strings among N components for each of which there exist A_z variants, is called the possibility space or design space of an artefact. The size S of the design space is given by:

$$S = A_1 \times A_2 \times \ldots \times A_N \qquad (1)$$

The K-value of a system refers to the number of interdependencies among components which are called "epistatic" relations. The ensemble of epistatic relations describe the system's internal structure. Epistatic relations between components imply that the functioning of one component is dependent both upon its own state and upon the state of K other components that impinge upon this component. For example, the functionality of an engine type relates to the cooling component used. The introduction of a more powerful engine type may require that the cooling system adapts in order to deal with increases in heat. The functionality of an engine type thus depends not only on the type of engine, but also on the type of cooling device. The majority of combinations between components create a malfunctioning or “imbalance”, while only few combinations result in a right fit (Rosenberg, 1969). The NK-model provides a formalism to analyse the complex interdependencies within a technological systems, and the evolutionary patterns that are expected to arise.

1.1 The NK-model

In the NK-model, the complexity of the internal structure is indicated by K, which stands for the number of components that affect the functioning of one component. The K-value is lowest when no epistatic relations exist ($K=0$), and highest when all components are epistatically related ($K=N-1$). Below, we discuss two explanatory simulations of two systems which both contain three components, but which differ in complexity ($K=0$ versus $K=2$).

Consider a technology ($N=3$, $A_z=2$) in which the components are functionally independent ($K=0$). The contribution of each component to the overall performance of the product is solely dependent upon its own state, which is either “0” or “1”. The contribution to the product’s functionality is called the fitness contribution of a component. Following Kauffman (1993), we draw the fitness values of components for the two possible states “0” and “1” randomly from an uniform distribution between 0.0 and 1.0. The fitness of the system as a whole F is calculated as the mean value of the fitness values of components with fitness meaning a summary measure of the performance level of a technology including their costs. *Figure 1* lists a simulation.

The distribution of fitness values of all possible strings is called the *fitness landscape* of a system, a concept which goes back to Wright (1932). The fitness landscape contains the fitness values for all coordinates in the design space. Analogous to one-step mutation in biological organisms, one can assume firms to search the design space by mutating one component at the time thus moving along one axis of the design space (“local search”). If mutation leads to a string with higher fitness, firms continue to search from there, while a new string with lower fitness induces a firm to return to the previous string, and continue to search from there. As long as there exist at least one neighbouring string that has a higher fitness, a firm can “climb” the fitness landscape by trial-and-error until it reaches an optimum (“hill-

$K=0$, the fitness landscape always contains only one optimum, which in our simulation is string 110 with F=0.80. No matter from which string a firm starts searching, it is always able to find the global optimum by a series of random one-component mutations.

In the case of maximum complexity, the functioning of components in a system depends upon all other components (*K=N-1*). The fitness contribution of each component is dependent upon the states of all other components. In this case, the value of the fitness contribution of each component has to be randomly drawn *for each combination of components separately*. Contrary to *K=0*-systems, the fitness landscapes of *K=N-1*-systems usually contain local optima, which are combinations of components that have a fitness value which cannot be improved by changing one component. Local optima are the consequence of interdependencies between components that render their functioning different in different combinations.

	f_1	f_2	f_3	F
000:	0.2	0.6	0.8	0.53
001:	0.2	0.6	0.5	0.43
010:	0.2	0.9	0.8	0.63
011:	0.2	0.9	0.5	0.53
100:	0.7	0.6	0.8	0.70
101:	0.7	0.6	0.5	0.60
110:	0.7	0.9	0.8	0.80
111:	0.7	0.9	0.5	0.70

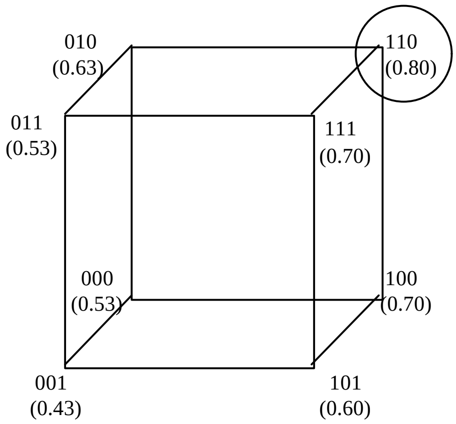


Figure 1: Fitness landscape of a N=3-artefact (K=0)

	f_1	f_2	f_3	F
000:	0.5	0.1	0.1	0.23
001:	0.3	0.5	0.3	0.37
010:	0.8	0.8	0.7	0.77
011:	0.3	0.6	0.6	0.50
100:	0.7	0.9	0.6	0.73
101:	0.2	0.4	0.7	0.43
110:	0.4	0.7	0.8	0.63
111:	0.5	0.8	0.9	0.73

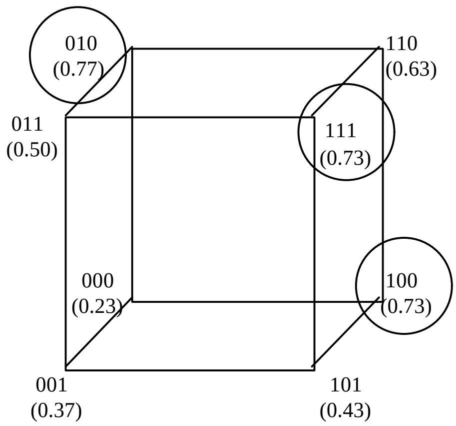


Figure 2: Fitness landscape of a N=3-technology (K=2)

An example of a possibility space of a *K=2*-system is given in *figure 2*. The fitness landscape contains three optima: 010, 100 and 111 thus rendering the fitness landscape “rugged”. String 010 is a global optimum since it has the highest fitness value, while strings 100 and 111 are local optima. For local optima it holds that firms occupying such a string cannot improve the fitness by one-component mutation, thus are unable to move to the global optimum. The one-component mutation algorithm can end up in different local optima depending on the initial starting point and the particular sequence of mutations that is followed. There is “path-dependency” in the search sequence of individual firms: two firms starting from the same initial design, may still end up in different local optima. For example, in the simulation in *figure 2*, a search starting from design 001 may end up in the local optimum 010 via combination 011, or in local optimum 111 via combination 101 or in local optimum 100 via combination 101. However, the industry as a whole is not likely to get locked into one technology: local search lead firms on different routes through the design space towards different local optima.

Importantly, the fitness values of optima lie close to one another, thus implying that several solutions are possible that are more or less equally fit. The difference between the fitness values of local optima tends to be smaller for systems with larger design spaces (which are typical for technological systems). Using the image of a landscape: complex systems have a fitness landscape with many peaks of about equal height. Thus, the existence of local opitmal designs that are equally fit, implies that technological variety can be sustained under selection.

1.2 Evolutionary dynamics at the firm and industry level

In economic evolution, there is “selection” at two levels of aggregation: trial-and-error learning within the firm which eliminates combinations that do not correspond with local optima, and market selection at the level of the industry which operates on designs that have been selected by firms. From the NK-model, one can derive that firms searching a complex space will not necessarily be able to find the global optimal string by means of a one-component mutation rule. Instead, they are expected to “lock-in” into locally optimal combinations. However, the analogies of biological evolution through one-gene mutation and environmental selection, and technological evolution through local search and market competition are in need of further substantive theorising. There are important differences in both the search algorithm and selection dynamics.

1.3 Economic meaning of fitness

When we accept the NK-model as a baseline model of technological evolution, the abstract notion of fitness should also be given a substantial meaning in economic terms. In principle, the fitness value of a technology *F* can be considered an overall measure of the performance of a technology relative to the costs of the components that are incorporated. In economic terms, fitness of a technology would then correspond to “value for money” (Saviotti 1996). We then can understand the concept of local optima in terms of technoeconomic trade-offs: one optimum in a landscape may correspond to an expensive, high-quality product incorporating expensive components, while another local optimum corresponds to a cheap, low-quality product incorporating less expensive components. The existence of such local optima likely leads a population of firms to *differentiate* only along an axis ranging from low-price, low quality product to high-price, high-quality products.

When technologies have multiple functions (size, speed, payload, etc.), one does not only expect price differentiation, but also product differentiation. In the latter case, designs corresponding to different local optima may well be equally costly, but these embody different trade-offs between selection criteria. Firm search in the possibility space is essentially directed to find a satisfactory set of trade-offs between functions which apply in a specific market segment.[1] Each coherent set of trade-off, i.e. each locally optimal combination between components, corresponds to a possible technological trajectory.

An example of different sets of trade-offs which lay down different technological trajectories in aircraft technology is the trade-off between engine power and wing surface: the development of the jet engine made higher speed levels possible, but the increased weight of the aircraft must be compensated by larger wings. Larger wings, however, diminish aerodynamics as well as the manoeuvrability of aircraft , the latter being of utmost importance in fighter operations. The development of jet engines called for an innovation in wings: delta wings compensate the increased weight without decreasing the aerodynamics and manoeuvrability. This led to a bifurcation into two trajectories where straight wings were coupled to low-power propeller engines used in low-speed transport operations, and delta wings were coupled to high-power jet engines for supersonic fighter operations. A third trajectory came into existence when turbofan engines were developed that have been coupled

[1] This idea has also been the basis for the simulation model of Windrum and Birchenhall (1998) using genetic algorithms.

to swept wings for medium-speed transport operations. This example illustrates that different technological trajectories embody different trade-offs between components in a system.

1.4 Hypotheses

From the previous discussion, we list the following hypotheses:

Hypothesis 1: the variety of designs is positively related to the complexity of a technology as reflected in the number of local optima in the design space. Different local optima reflect different sets of trade-offs between the functional criteria of a technology (including its costs) thus allowing for market segmentation to occur

Hypothesis 2: The degree of specialisation among firms is positively related to the complexity of a technology as reflected in number of local optima in the design space. The path-dependent nature of problem-solving imply that different firms are likely to find different local optima. Due to costs of switching to alternative designs, choice of design is sticky: once a firm has found a locally optimal design, it is expected to continue to develop new technologies according this one specific design.

2. Entropy statistics

Entropy statistics and information theory are tools for analysing complex, distributed systems. The concept of entropy is used here to study variety patterns in technological evolution on the basis of the NK-model. Though the concept of entropy originated from thermodynamic systems, it has acquired a general probabilistic meaning that allows for a large number of applications (Theil 1969, 1972; Langton 1990). Entropy statistics is based solely on the properties of probability distributions, and, as such, is especially suitable for studying evolutionary phenomena at the level of any population of heterogeneous entities (Saviotti 1996). In this study, we are interested in the empirical relation between variety and technological complexity (hypothesis 1) and the relation between firms' degree of specialisation and technological complexity (hypothesis 2), which can be expressed in entropy statistical terms.

2.1 Hypothesis 1

In the NK-model above, we represented technologies as a set of components. More generally, components can be considered dimensions along which entities can differ, and thus need not to refer only to physical components, but rather to any variable in which technologies are described. For example, in the following, the number of engines within a technologies is also taken as a dimension along which design can differ. The dimensions are labelled X_1 , X_2 , …, X_N. Each technology can thus described by a N-dimensional string, and at each moment in time, one can constitute a N-dimensional frequency distribution (*z=1,..,N*), which represents the population of technologies. Recall that for each dimension we have A_z component variants, and since we write for the first variant a "0", the second variants a "1", etc., we have for the first dimensions $i=0,\ldots,A_1-1$, for the second dimension $j=0,\ldots,A_2-1$, etc.. The entropy value of a N-dimensional distribution is given by (Theil 1972):

$$H\,(X_1, X_2, \ldots, X_N) = -\sum_{i=0}^{A_1-1}\ \sum_{j=0}^{A_2-1}\cdots\sum_{w=0}^{A_N-1}\left(p_{ij\ldots w}\cdot\log_2 p_{ij\ldots w}\right) \qquad (2)$$ [2]

in bits (since we use logarithm two). Entropy is a measure of variance or uncertainty: the higher the entropy, the more difficult it is to predict the design of a technology which is blindly picked from the population. The entropy is zero when all designs are equal since then there is no uncertainty, and is positive otherwise. The larger the entropy value, the larger the technological variety within a distribution of designs. The maximum entropy is limited by the design space *S* (see formula 1): when all *S* possible combinations $p_{ij\ldots w}$ have an equal frequency, we have $p_{ij\ldots w} = 1 / (A_1 \times A_2 \times \ldots \times A_N)$. The entropy of this distribution equals $H = \log (A_1 \times A_2 \times \ldots \times A_N)$ which is its maximum possible value (Theil 1972)

The first hypothesis states that the variety of designs as indicated by the entropy of the frequency distribution is positively related to the number of local optima. The presence of local optima would imply that designs that have a relatively high frequency in the population, differ in at least two components, since local optima have been defined as combinations between components that cannot be improved by mutating one component. The larger the

---

[2] For $0 \times {}^2\log 0 \equiv 0$.

extent in which specific variants of one component are coupled to specific variants of other components, the larger the number of local optima. The degree of this coupling or dependence within a multidimensional distribution is measured by what is called the mutual information T. The N-dimensional mutual information is given by:

$$T(X_1, X_2, \ldots, X_N) = \sum_{i=0}^{A_1-1} \sum_{j=0}^{A_2-1} \ldots \sum_{w=0}^{A_N-1} \left(p_{ij..w} \cdot \log_2 \left(p_{ij...w} / \left(p_{i.....} \cdot p_{.j....} \cdot \ldots \cdot p_{.....w} \right) \right) \right) \quad (3)$$

in bits. The mutual information value equals zero when there is exist no coupling/dependence between any of the dimensions, and the higher the mutual information value the higher the degree of coupling. Hypothesis 1 thus states that a rising entropy co-occurs with a rising mutual information and a falling entropy co-occurs with a falling mutual information.

Example

To illustrate that the mutual information measures the degree of coupling between components which reflect the presence of local optima, consider again the simulation of a complex *N=3*-system in *figure 2*, in which combinations 010, 100 and 111 were the local optima.

Case 1. Imagine that all technologies offered on the market are designed according to one out of two optima 010 and 100, and at equal frequency. We have for the three-dimensional frequencies p_{010}=0.50 and p_{100}=0.50, and zero frequencies for the other possible combinations. The variety in the population as measured by the entropy of the distribution, adds up to:

$$H(X_1,X_2,X_3) = - (0.50 \cdot \log_2 (0.50)) - (0.50 \cdot \log_2 (0.50)) = 1.00 \text{ bit}$$

To determine the mutual information, we calculate univariate frequencies which are $p_{0..}$=0.50, $p_{1..}$=0.50, $p_{.0.}$=0.50, $p_{.1.}$=0.50, $p_{..0}$=1.00, and $p_{..1}$=0.00. The $T(X_1,X_2,X_3)$-value becomes:

$$T(X_1,X_2,X_3) = (0.50 \cdot \log_2 (0.50/0.25)) + (0.50 \cdot \log_2 (0.50/0.25)) = 1.00 \text{ bit}$$

Case 2. Now, imagine that the third local optimum 111 is found and that the number of products that are developed according to this locally optimal design is equal to the number of products that are designed according to the other two locally optimal designs. We have for the three dimensional frequencies p_{010}=0.33, p_{100}=0.33, and p_{111}=0.33. The entropy becomes:

$$H(X_1,X_2,X_3) = - (0.33 \cdot \log_2 (0.33)) - (0.33 \cdot \log_2 (0.\ 33) - (0.33 \cdot \log_2 (0.33)) = 1.58 \text{ bits}$$

Comparing the entropy (variety) in *case 2* with the entropy (variety) in *case 1*, we find that the variety has increased from *case 1* to *case 2*.

To calculate the mutual information for *case 2*, we have for the univariate frequencies $p_{0..}$=0.33, $p_{1..}$=0.67, $p_{.0.}$=0.33, $p_{.1.}$=0.67, $p_{..0}$=0.67, and $p_{..1}$=0.33. The mutual information $T(X_1,X_2,X_3)$ becomes:

$$T(X_1,X_2,X_3) = (0.33 \cdot \log_2 (0.33/0.15)) + (0.33 \cdot \log_2 (0.33/0.15)) + (0.33 \cdot \log_2 (0.33/0.15)) = 1.17 \text{ bits}$$

The mutual information is *case 2* is thus greater than in *case 1* reflecting that the number of local optima that are occupied within the population has increased from *case 1* to *case 2*.

Case 3. The mutual information is not necessarily positively related to the entropy (if it would be the case, hypothesis 1 would always hold). For example, consider the case that there exist again three combinations with equal frequency as in *case 2*, but these combinations concern 010, 100 and 110 (instead of 111 as in *case 2*), so we get for the three dimensional frequencies p_{010}=0.33, p_{100}=0.33, and p_{110}=0.33. The entropy is the same as in *case 2*, since:

$$H(X_1,X_2,X_3) = - (0.33 \cdot \log_2 (0.33)) - (0.33 \cdot \log_2 (0.\ 33)) - (0.33 \cdot \log_2 (0.33)) = 1.58 \text{ bits}$$

The difference between this case and *case 2* is that the three combinations with positive frequency in this case cannot correspond all to local optima, since combination 110 is only different with respect to one component from combination 010 and 100. To calculate the mutual information, we get for the univariate frequencies $p_{0..}$=0.33, $p_{1..}$=0.67, $p_{.0.}$=0.33, $p_{.1.}$=0.67, $p_{..0}$=1.00, and $p_{..1}$=0.00. The mutual information $T(X_1,X_2,X_3)$ becomes:

$$T(X_1,X_2,X_3) = (0.33 \cdot \log_2 (0.33/0.22)) + (0.33 \cdot \log_2 (0.33/0.22)) + (0.33 \cdot \log_2 (0.33/0.44)) = 0.25 \text{ bit}$$

which is much lower than the values for *case 1* and *case 2*.[3]

2.2 Hypothesis 2

The above entropy measure is applied to the distribution of designs offered within an industry in a given period of time. To determine the degree of specialisation of firms, we can repeat the measurement of entropy for the distribution of designs developed by each single firm b in a given period of time. We then get for the entropy value of the distribution at the level of the firm b:

$$H_b(X_1, X_2,..., X_N) = - \sum_{i=0}^{A_1-1} \sum_{j=0}^{A_2-1} ... \sum_{w=0}^{A_N-1} \left(p_{ij...wb} \cdot \log_2 p_{ij....wb} \right) \qquad (4)$$

in bits. The average entropy of technologies within firms weighted for their relative share, called the weighted entropy H_B is given by:

$$H_B(X_1, X_2,..., X_N) = \sum_{b=1}^{B} \left(p_b \cdot H_b(X_1, X_2,..., X_N) \right) \qquad (5)$$

in bits. where p_b stands for the relative number of technologies developed by firm b in the industry for a total number of B firms. It can be shown, that this weighted sum of the firms' entropy values (formula 5) cannot exceed the total entropy at the level of industry as given in formula (2) (Theil 1972). The difference between H and H_B is known as a measure of segregation and indicates the degree of specialisation at some level of decomposition, which in this case is the firm level (Theil 1972). We get for the value of the measure of segregation/specialisation S_B:

$$S_B = H(X_1, X_2,..., X_N) - H_B(X_1, X_2,..., X_N) \qquad (6)$$

[3] It can be shown that the mutual information always equals zero when the entropy is minimum ($H=0$), and that the mutual information is always zero when entropy is maximum ($H=log(A_1 \times A_2 \times ... \times A_N)$). For entropy values in between zero and its maximum value, mutual information is zero or positive, with the exact value depending on the degree of dependence among the dimensions (Theil 1972; Langton 1990).

in bits. The segregation measure indicates the extent in which the entropy of designs at the industry level differs from the entropy within the firms. When each firm would develop its technologies according to one single design trajectory, H_b would equal zero for all b firms, and thus H_B would equal zero too. In that case, the specialisation measure S_B takes on its maximum value ($S_B = H$), indicating perfect specialisation of firms. And, when the distribution of designs of each firm would correspond to the distribution of technologies at the industry level, H_B would equal H, and the specialisation measure S_B takes on its maximum value ($S_B = 0$). Hypothesis 2 thus states that a rising segregation/specialisation of firms co-occurs with a rising mutual information, and a falling segregation/specialisation among firms co-occurs with a falling mutual information.

3. Results

The data concern technical dimensions of aircraft, helicopters and motorcycles, which are listed in the *Appendix 1*.[4] Each design is thus coded as a string of length N describing its components variants ($N=6$ for aircraft, $N=5$ for helicopters and $N=3$ for motorcycles). For example, the Boeing 747 incorporates a turbofan, four engines, monoplane, swept wing, one boom and one tail, and is described by string 332000.

The period that is covered by the data is *1913-1984* for aircraft technologies, *1940-1983* for helicopter technologies and *1911-1995* for motorcycle technologies. This data-material thus allows for a long-term analysis of their evolution. To constitute a frequency distribution for each period in time, on the basis of which entropy statistics are calculated, we have chosen ten-year periods as to assure a sufficient number of observations in each period.[5] In the following figures, the year on the x-axis refers to the last year of a ten year period (for example, 1909 refers to the period 1900-1909, 1910 refers to the period 1901-1910, etc.). Note that the frequency distribution concerns the distribution of designs *offered* on the market, and not the frequency in terms of their sales. Put another way, we consider the technological evolution in terms of the distribution of designs offered on the market, and not of economic evolution in terms of the distribution of products sold on the market.

---

[4] The data used here are the same as those used in a previous study (Frenken, Saviotti, Trommetter, forthcoming). The data on microcomputers which had been used in the previous study, have not been used here since this database lacks the information on the firm offering the technology.

The results on the mutual information, entropy and segregation values are all listed in one graph for each technology. The results for aircraft are listed in *graph 1*, the results for helicopters in *graph 2* and the results for motorcycles in *graph 3*.

3.1 Hypothesis 1: Results on entropy and mutual information

For all three technologies analysed here, we observe that in general the trend in entropy follows the trend in mutual information. For aircraft and helicopters, this relation holds for the whole period, while for motorcycles we find this relationship only for the period (1955-1985). Thus, run hypotheses 1 generally holds: the degree in which a population of products is distributed among local optima as indicated by the mutual information relates positively with the design variety as indicated by the entropy.

Though the trends in mutual information and entropy are positively related, the direction of these trends are different. In the case of aircraft and motorcycles values are mainly rising (with values for motorcycles highly fluctuating due to the limited number of observations). The rising trends in aircraft and motorcycles indicate that the number of locally optimal designs and corresponding market segments has grown over time. By contrast, the trends in helicopters are first rising and then falling again. The recent decrease in values for helicopters indicate a fall in the number of locally optimal designs and consequently, in the variety of design offered on the market (cf. Frenken, Saviotti, Trommetter, forthcoming).

To look which locally optimal designs are relatively frequent, we can rank for each design combination $p_{ij\ldots w}$ its value for $p_{ij\ldots w}\ log2\ \ (p_{ij\ldots w} / p_{i\ldots\ldots} \times p_{.j\ldots.} \times \ldots \times p_{\ldots..w})$ out of which the mutual information is made of (*formula 3*). The highest values indicate the design combinations *ij…w* that are contributing to the largest extent to the mutual information value. In the history of aircraft, four combinations show a particular high value:

000000: piston-propeller, one-engine, straight-wing monoplane with one boom and one tail
110000: turboprop, two-engine, straight-wing monoplanes with one boom and one tail
211000: jet, two-engine, delta-wing monoplane with one boom and one tail
332000: turbofan, four-engine, swept-wing monoplane with one tail and one boom

---

[5] In Frenken, Saviotti, Trommetter (forthcoming), entropy was based on periods with an equal number of observations instead of an equal number of years. Periods with an equal number of observations were chosen to compare entropy with an alternative measure (Weitzman 1992), which is sensitive to the number of observations.

Each of these four designs is primarily used in specific market segments: 000000 in small, low-speed business and trainer aircraft, 110000 in medium-sized, low-speed transport aircraft, 201010 in small, supersonic fighter aircraft and 332000 in large, medium-speed aircraft.

In the history of helicopters, we find high values for the following three combinations:

00000: piston, one-engine, two-blades, one-shaft and one rotor per shaft
41200: turboshaft, two-engine, four-blades, one-shaft and one rotor per shaft
41410: turboshaft, two-engine, six-blades, two-shaft and one rotor per shaft

The 00000-design correspond to low-payload, short-range helicopters, the 41200-design to medium-payload, long-range helicopters and 41410-design to high-payload, short-range helicopters. From the sixties onwards, the relative number of product developments in designs 00000 and 41410 decreased, and the relative number of product developments in design 41200 increased. This latter design is used for medium-payload and long-range, and has become the "dominant design" (cf. Abernathy and Utterback, 1978). The decreasing trends in entropy and mutual information indicate the dominance of this single technology trajectory, but it should be noted that the two alternative designs have always been present throughout the whole history, but in decreasing numbers.

In the history of motorcycles, we find high values for combinations:

031: two-stroke, four-cylinder engine cooled by water
110: four-stroke, two-cylinder engine cooled by air
141: four-stroke, four-cylinder engine cooled by water

The four-stroke, two-cylinder motorcycles cooled by air (110) are most frequent throughout the history of motorcycles. This design can be considered the "dominant design", notably since the success of the Triumph Twin Speed introduced in 1937 (Brown, 1996). The second trajectory which emerged only in the seventies concern high-speed racing models with a two-stroke, four-cylinder engine cooled by water (031). A third trajectory which also emerged around the seventies concern heavy touring models with a four-stroke, six-cylinder engine cooled by water (141).

3.2 Hypothesis 2: Results on segregation and mutual information

We observe a general, positive relation between changes in mutual information and the changes in segregation, again with the exception of some periods in motorcycle history. Thus, when mutual information increases, segregation values tend to increase accordingly, and *vice versa*. Hypothesis 2 thus holds at least for aircraft and helicopters: the number of local optima as reflected by the value of mutual information is a determinant of the degree in which firms can specialise along different trajectories. In aircraft where the number of local optima and corresponding market segments increased, we observe a increasing degree of specialisation among firms. In the history of helicopter technology where the number of local optima first increased and then decreased, the degree of specialisation of firms first increased and then decreased.

The number of local optima as indicated by the mutual information thus seems to determine the scope of specialisation among firms. When this number is rising, firms can specialise on specific design trajectories thus exploiting dynamic economies related to sustained user relations and production efficiencies. The nature of competition is then expected to change from predominantly price competition to from predominant monopolistic competition as the market is increasingly segmented in niches (cf. Saviotti 1996).

Summarizing, in all three technologies we observed a relationship between complexity as indicated by the mutual information on the one hand, and entropy (variety) and segregation (specialisation) on the other hand. The mutual information points to different market segments in which specific designs dominate, and in which firms tend to specialise.

4. Concluding remarks

We started out by outlining a model of technological evolution which takes into account the complex interdependencies between the components of technologies. Different locally optimal design solutions exist that embody different trade-offs between different functions of a technology. Consequently, firms exploiting one local optima are likely to specialise in the corresponding market segment. We derived two hypotheses: first, the technological variety that can be sustained in a market is positively related to the complexity of a technology, and second, the degree of specialisation among firms is positively related to the degree of complexity of a technology. By comparing the entropy of a distribution with the mutual information, the first hypothesis has been tested, and evidence was found for aircraft and helicopters, and limited evidence for motorcycles. The comparison of segregation with mutual information tested the second hypothesis, which again held for the whole history of aircraft and helicopters, but only for a certain period in the history of motorcycles.

The NK-model and entropy methodology lay out a number of possible future research opportunities which possibly contribute to bridging the gap between formal modelling and empirical analysis in evolutionary economics. First, there is the question of (nearly)-decomposability of large, complex systems into subsystems which reduce search costs in vast design spaces (Simon 1969). This issue has been treated analytically in Frenken, Marengo and Valente (forthcoming). Empirical analysis of sub-systemic structures require more detailed descriptions of technologies in terms of their variables. A first attempt to find decomposable structures in large systems is reported in Frenken, Marengo, Valente (1999) where eight variables of over 4000 microcomputers have been analysed. Second, the NK-model can be applied outside the domain of technological evolution to various systems that are characterised by interdependencies in their functioning. Levinthal (1997) and Marengo (1998) have started to explore the logic of the NK-model in the light of the evolution of firms, and Frenken (1998) applied the NK-model to networks of producers, users and governments.

Model of complex systems are, in principle, content-free, and they can be applied to many domains. However, each application is in need of substantive theoretical specification and, ideally, should specify variables which are empirically identifiable.

References

Abernathy, W., Utterback, J. (1978), Patterns of industrial innovation, *Technology Review* 50, 41-47.

Arthur, W.B. (1989), Competing technologies, increasing returns, and lock-in by historical events, *Economic Journal* 99, 116-131.

Dosi, G. (1982), Technological paradigms and technological trajectories. A suggested interpretation of the determinants and directions of technical change, *Research Policy* 11, 147-162.

Fisher, R.A. (1930), *The Genetical Theory of Natural Selection* (Oxford: Clarendon Press).

Frenken, K. (1998) ‘Modelling the triple helix using complex systems theory’, *Paper presented at the Second Symposium on the Triple Helix of University-Industry-Government Relations*, New York, January 4-6.

Frenken, K., Marengo L., Valente, M. (forthcoming) ‘Interdependencies, nearly-decomposability and adaptation’, in: Brenner, T. (ed.) *Computational Techniques to Model Learning in Economics* (Boston etc., Kluwer).

Frenken, K., Marengo L., Valente, M. (1999) ‘Complexity and nearly-decomposability in technological fitness landscapes. Theoretical implications and an empirical application to PC technology’, *manuscript*, February.

Frenken, K., Saviotti, P.P., Trommetter, M. (forthcoming) ‘Variety and niche creation in aircraft, helicopters, motorcycles and microcomputers’, *Research Policy*.

Kauffman, S.A. (1993) *The Origins of Order. Self-Organization and Selection in Evolution* (New York etc.: Oxford University Press).

Kauffman, S.A., Macready, W. (1995), Technological evolution and adaptive organizations, *Complexity* 1, 26-43.

Langton, C.G. (1990), Computation at the edge of chaos, *Physica D* 42, 12-37.

Levinthal, D. (1997), Adaptation on rugged landscapes, *Management Science* 43, 934-950.

Marengo, L. (1998), Interdependencies and division of labour in problem-solving technologies, *Seventh Conference of the International Schumpeter Society*, Vienna, 13-16 June.

Nelson, R.R., Winter, S.G. (1982) *An Evolutionary Theory of Economic Change* (Cambridge MA & London: Belknap Press of Harvard University Press).

Rosenberg, N. (1969), The direction of technological change: inducement mechanisms and focusing devices, *Economic Development and Cultural Change* 18, 1-24 (reprinted in: *Perspectives on Technology*, Cambridge University Press, Cambridge, 1976, 108-125).

Saviotti, P.P. (1996) *Technological Evolution, Variety and the Economy* (Cheltenham: Edward Elgar).

Simon, H.A. (1969), *The Sciences of the Artificial* (Cambridge MA & London: MIT Press).

Theil, H. (1969), *Economics and Information Theory* (Amsterdam: North-Holland).

Theil, H. (1972) *Statistical Decomposition Analysis* (Amsterdam: North-Holland).

Weitzman, M.L. (1992) 'On diversity', *Quarterly Journal of Economics* 107, 363-406.

Wright, S. (1932) 'The roles of mutation, inbreeding, crossbreeding and selection in evolution', *Proceedings of the Sixth International Congress of Genetics* 1, 356-366.

Windrum, P., Birchenhall, C. (1998), Is product life-cycle theory a special case? Dominant designs and the emergence of market niches through coevolutionary learning, *Structural Change and Economic Dynamics* 9, 109-134.

Appendix 1

Description of aircraft, helicopter and motorcycle data

Aircraft

Source: Jane's (1989) *Jane's Encyclopedia of Aviation* (London: Studio Editions)

Number of observations: 731

Time span: 1913 - 1984

Variable labels (N = 6):

X_1 : Engine type
X_2 : Number of engines
X_3 : Wing type
X_4 : Number of wings
X_5 : Number of booms
X_6 : Number of fins

Value labels:

Engine type ($A_1 = 5$):	Piston propeller (“0”), Turboprop (“1”), Jet (“2”), Turbofan (“3”), Rocket (“4”)
Number of engines ($A_2 = 6$):	One (“0”), Two (“1”), Three (“2”), Four (“3”), Six (“4”),
Wing type ($A_3 = 4$):	Straight (“0”), Delta (“1”), Swept (“2”), Variable swept
Number of wings ($A_4 = 3$):	Monoplane (“0”), Biplane (“1”), Triplane (“2”)
Number of booms ($A_5 = 3$):	One (“0”), Two (“1”), Three (“2”)
Number of tails ($A_6 = 2$):	One (“0”), Two (“1”)

Helicopter

Source: Jane's (1989) *Jane's Encyclopedia of Aviation* (London: Studio Editions)

Number of observations: 144

Time span: 1940 - 1983

Variable labels (N = 5):

X_1 : Engine type
X_2 : Number of engines
X_3 : Number of blades
X_4 : Number of shafts
X_5 : Number of rotors per shaft

Value labels:

Engine type ($A_1 = 5$):	Piston (“0”), Piston turbo (“1”), Ramjet (“2”), Gas generator (“3”), Turboshaft (“4”)
Number of engines ($A_2 = 3$):	One (“0”), Two (“1”), Three (“2”),
Number of blades ($A_3 = 7$):	Two (“0”), Three (“1”), Four (“2”), Five (“3”), Six (“4”), Seven (“5”), Eight (“6”)
Number of shafts ($A_4 = 2$):	One (“0”), Two (“1”)
Number of rotors per shaft ($A_5 = 2$):	One (“0”), Two (“1”)

Motorcycle

Source: Brown, R. (1996) *Encyclopédie de la Moto* (London: Anness Publishing Ltd.)

Number of observations: 80

Time span: 1911 - 1995

Variable labels (N = 3):

X_1 : Engine type

X_2 : Number of cylinders

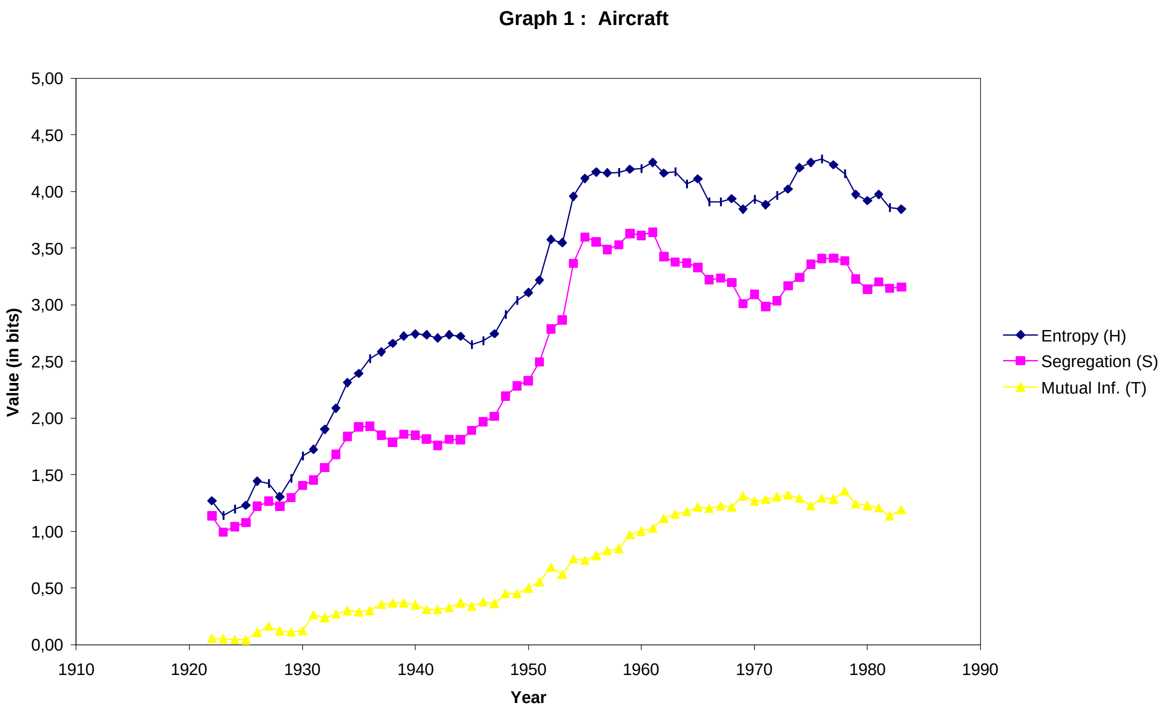
X_3 : cooling system

Value labels:

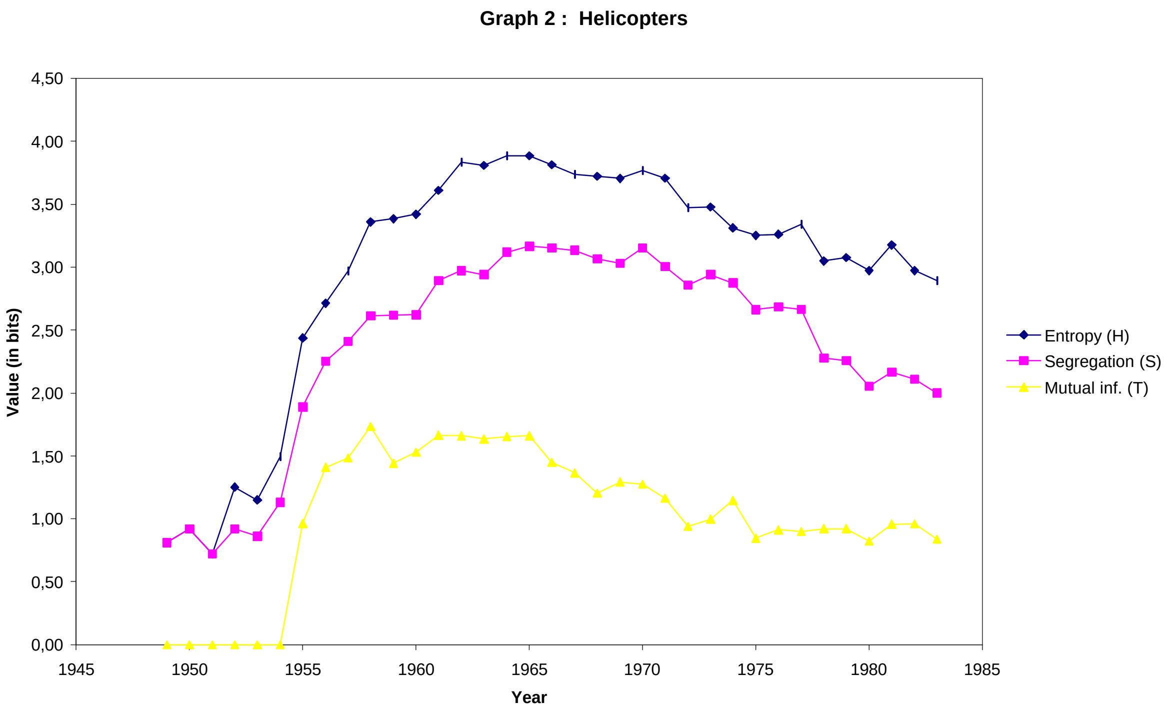
Engine type ($A_1 = 2$):	Two-stroke (“0”), Four-stroke (“1”)
Number of cylinders ($A_2 = 5$):	One (“0”), Two (“1”), Three (“2”), Four (“3”), Six (“4”)
Cooling system ($A_3 = 3$):	By air (“0”), By water (“1”), By oil (“2”)

Appendix 2

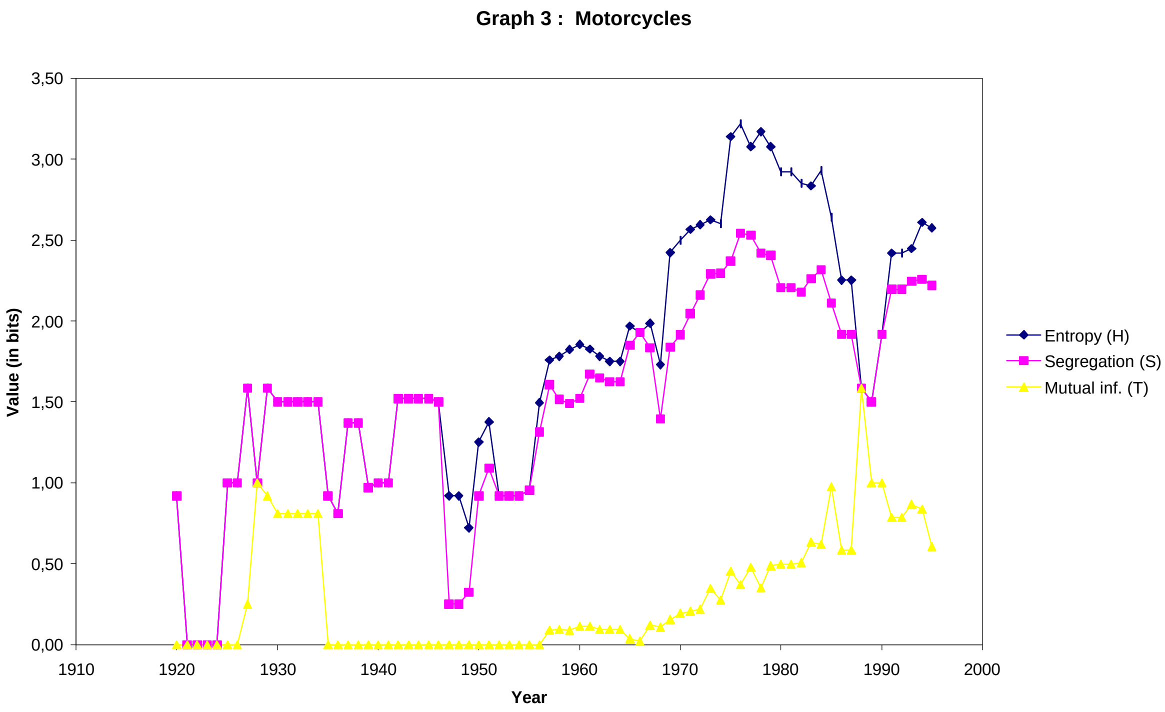
Graphs (1-3)



Graph 1 : Results for aircraft



Graph 2: Results for helicopters



Graph 3: Results for motorcycles